Measuring the Luttinger liquid parameter with shot noise

J. K. Kühne,[1,2] I. V. Protopopov,[2,3] Y. Oreg,[4] and A. D. Mirlin[1,2,5]

[1] Institut für Nanotechnologie, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany
[2] Institut für Theorie der Kondensierten Materie,
Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany
[3] L. D. Landau Institute for Theoretical Physics RAS, 119334 Moscow, Russia
[4] Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot, Israel 76100
[5] Petersburg Nuclear Physics Institute, 188300 St. Petersburg, Russia.

We explore the low-frequency noise of interacting electrons in a one-dimensional structure (quantum wire or interaction-coupled edge states) with counterpropagating modes, assuming a single channel in each direction. The system is driven out of equilibrium by a quantum point contact (QPC) with an applied voltage, which induces a double-step energy distribution of incoming electrons on one side of the device. A second QPC serves to explore the statistics of outgoing electrons. We show that measurement of a low-frequency noise in such a setup allows one to extract the Luttinger liquid constant K which is the key parameter characterizing an interacting 1D system. We evaluate the dependence of the zero-frequency noise on K and on parameters of both QPCs (transparencies and voltages).

I. INTRODUCTION

The physics of interacting electrons in one dimension (1D) is profoundly different from that in higher dimensions. It is well known that the correspondence between interacting electrons and free fermionic quasiparticles, which is in the core of Landau's Fermi-liquid theory, breaks down in 1D. The resulting strongly correlated state, known as the Luttinger liquid (LL), can not be treated by conventional Fermi-liquid methods. Fortunately, there exists an extremely powerful approach to the problem, the bosonization technique[1–6]. It describes the low-energy sector of the theory in terms of density fluctuations, which are, under the simplest circumstances, non-interacting bosons.

A key parameter invoked in the bosonization description of a LL state is the interaction constant K. This dimensionless parameter gives an effective measure of the strength of the interaction between the electrons, with $K = 1$ corresponding to a non-interacting Fermi gas, $K < 1$ to repulsion, and $K > 1$ to attraction. The LL constant K controls the behavior of various physical properties of the system[3], including, e.g., the scaling of the tunneling density of states away from the wire ends (TDOS)[7], $\nu(\epsilon) \propto |\epsilon|^{(1-K)^2/2K}$, the temperature-dependence of the conductance through a tunnel barrier in a Luttinger liquid[8], $G(T) \propto T^{2(1-K)/K}$, and the temperature scaling of the conductivity of a disordered interacting wire[9]. There exists by now a rich variety of experimental realizations of LLs with fermionic constituent particles, including semiconductor, metallic, and polymer nanowires[10], carbon nanotubes[11], edge states of 2D topological insulators[12], and cold-atom systems[13].

Further, edges of quantum Hall systems[14–16] give rise to chiral LLs with only one propagation direction. When two such edges with opposite chirality are coupled by interaction, an artificial "wire" emerges[17,18]. Properties of LL structures are probed in a growing number of sophisticated experiments, in particular under strongly non-equilibrium conditions. A quantitative interpretation of experimental findings requires the knowledge of the LL parameter K of the studied system by an additional independent measurement of K.

Let us consider a typical experimental setup where a 1D conductor is connected to the outside world by leads. One possibility to access the value of K is to measure the power-law behavior of the TDOS by exploring the tunneling into the Luttinger liquid. This requires, however, an introduction of an additional probe to the interacting wire and is not simple experimentally. In addition, the tunneling characteristics may be affected by the interaction of the wire with the environment[19]. We can ask if it is possible to infer K considering the LL as a "black box" (in the spirit of scattering theory of electronic conduction[20]) and performing electrical measurements in the leads alone. One could naively expect that the interaction inside the system modifies the conductance of the wire, thus providing a direct experimental way to measure K. It is not correct, however. It is well known[21–24] that, due to absence of fermionic backscattering in a clean LL, its DC conductance is given by the interaction-independent value e^2/h. Moreover, while under generic non-equilibrium conditions the distribution function of the electrons that have passed the interacting part of the system depends on the interaction strength[25], the zero-frequency full counting statistics of the charge transferred through the system[26] is insensitive to the interaction. On the other hand, the non-equilibrium noise[27] and the full counting statistics[26] at high frequencies (of the order of or larger than the inverse flight-time through the system) do depend on the interaction strength but they are challenging to measure experimentally[28].

In this work we show that the interaction in a LL wire

can, however, be probed by low-frequency charge noise measurements provided that the electrons emerging from the LL are mixed (via scattering at an additional quantum point contact, QPC) with electrons coming from an independent reservoir. A similar approach was proposed recently to probe the (pseudo-)spin-charge separation in systems of co-propagating channels[29].

The structure of the paper is as follows. In Sec. II we introduce a device, consisting of 4 sources (SL, SR, S1, S2) and 2 drains D1 and D2 that are connected by two point contacts characterized by transmission and reflection coefficients t^2 and r^2 (see Fig 1). The system is driven out of equilibrium by an"injection" of electrons with double-step energy distribution trough the source SR. Such a distribution may be naturally prepared by means of an additional QPC0 (not shown in Fig. 1). The step height h is then given by its transmission coefficient. In Sec. III we calculate the shot noise in drain D2 as a function of the Luttinger liquid parameter K. Section III A is devoted to description of the general formalism while Sec. III B summarizes the results in the limits of weak ($|K-1| \ll 1$) and strong ($K \ll 1$) interaction for voltage U in SL much larger than the inverse of the flight time τ_l in the interaction region. In Sec. III C we present few numerical results for generic values of interaction parameter K.

Specifically, Figs. 5 and 6 illustrate the central results of the paper. Figure 5 demonstrates the dependence of the noise at zero voltage V (at source $S2$) on the parameter h of the double-step distribution (1) of incoming right-moving electrons. The noise attains its maximal value when the initial distribution is particle-hole symmetric ($h = 0.5$) and the QPC mixing the electrons from the LL wire with those from the source S2 has reflection probability $r^2 = t^2 = 0.5$. The ratio of this maximal noise to the voltage U in SL is a universal function of the LL parameter. Fig. 6 shows that the maximal current noise at drain D2 at zero frequency and zero voltage in source S2 ($\omega = 0$ and $V = 0$), which we denote as $\max_{h,r^2,t^2} [S_{D2}(\omega = 0, V = 0)]$, provides a direct access to the value of the LL parameter K. Although we do not have a simple analytic expression for the noise in this situation, the curve is universal and our numerical results can be used to determine K.

Section IV presents the calculation of the noise in drain D1 and Sec. V discusses the situation for attractive interactions. We conclude the manuscript with a summary section, Sec. VI.

II. SETUP

A setup that we consider in the present paper (and that is particularly relevant in the context of quantum Hall physics, see, e.g., Ref. 18) is shown in Fig. 1. It includes two counterpropagating electronic (or, more generally, fermionic) modes, right-movers R and left-movers L, interacting over a distance l via a short-range interac-
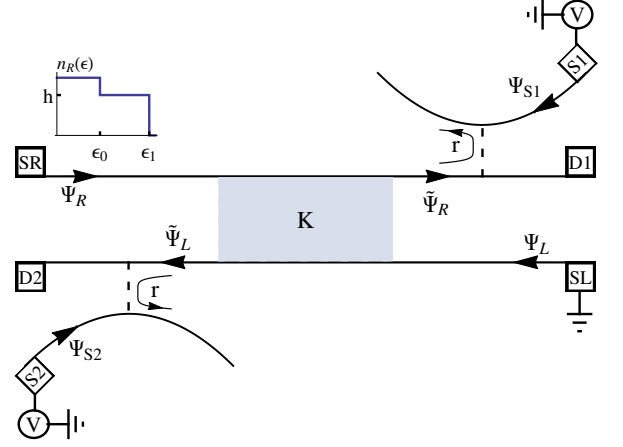

FIG. 1: Setup. Incoming R-electrons have a non-equilibrium double-step energy distribution (1) characterized by the step width $U = \epsilon_1 - \epsilon_0$ and height h. This distribution may be prepared by means of a QPC0 (not shown). The parameter h is given in this case by the transmission probability of the QPC0, while the parameter U is the QPC0 voltage. Incoming L-electrons, as well as $S1$- and $S2$-electrons are at equilibrium but the distribution of the $S1$ and $S2$ electrons can be tuned with a voltage V.

tion characterized by the LL parameter K. The system is driven out of equilibrium by an"injection" of incoming R-electrons (source SR) with a double-step energy distribution,

$$n_R(\epsilon) = (1 - h)n_0(\epsilon - \epsilon_0) + hn_0(\epsilon - \epsilon_1), \qquad (1)$$

Here, $n_0(\epsilon) = \Theta(-\epsilon)$ is the zero-temperature Fermi-Dirac distribution with zero chemical potential and $\epsilon_0 = -hU$, $\epsilon_1 = (1 - h)U$ are the positions of the Fermi edges[30]. The double-step distribution (1) may be naturally prepared by means of a QPC0 (not shown in Fig. 1). The parameter h is given in this case by the transmission probability of the QPC0, while the parameter U is the QPC0 voltage. (We set the electron charge e to unity throughout the paper, restoring it in the final expressions only.) The left-moving mode starts at zero temperature and zero voltage from the source SL. After traversing the interacting part of the wire, right-movers and left-movers are mixed with electrons from sources S1 and S2 (kept at zero temperature and chemical potential V) via scattering at QPCs with transmission (reflection) amplitudes t (r). We are interested in the charge noise at drains D1 and D2

$$S_{D1/D2}(\omega, V) = \int_\infty^\infty dt \left\langle \left\{ \delta I_{D1/D2}(t), \delta I_{D1/D2}(0) \right\} \right\rangle e^{-i\omega t},$$
$$(2)$$

where δI_{Di} its the fluctuating part of the current operator at the drain i, and curly brackets denote the anticommutator. The second argument of $S_{D1/D2}$ in Eq. (2) emphasizes the dependence of noise on the voltage V applied to the sources S1 and S2.

III. NOISE AT DRAIN D2

A. General formalism

We start with the discussion of the noise at the drain D2, which has a simpler structure. To evaluate S_{D2}, we express the electronic operator Ψ_{D2} at the drain D2 in terms of the fermionic fields at source S2 and at the output of the wire (see Fig. 1),

$$\Psi_{D2} = t\tilde{\Psi}_L + r\Psi_{S2}. \tag{3}$$

Together with the expression for the current operator, $I_{D2} = ue\Psi_{D2}^+\Psi_{D2}$ (where u is the velocity of the left fermionic mode and $e > 0$ is the absolute value of electron charge), this implies the decomposition of $S_2(\omega, V)$ into three contributions,

$$S_{D2} = |r|^4 S_{D2}^{(S2)} + |t|^4 S_{D2}^{(SL)} + |r|^2|t|^2 S_{D2}^{(\text{x})}. \tag{}$$

The first contribution to the sum represents just the charge noise of zero-temperature non-interacting electrons coming from the source S2, $S_{D2}^{(S2)}(\omega) \equiv S_0(\omega) = e^2|\omega|/2\pi$.

The key point for the discussion of the second contribution $S_{D2}^{(SL)}$ (which is also V-independent and represents the charge noise for fully open QPC) is the observation that it can be expressed solely in terms of the electronic density $\tilde{\rho}_L \equiv \tilde{\Psi}_L^+\tilde{\Psi}_L$, $S_{D2}^{(SL)}(t) = u^2 \langle\{\tilde{\rho}_L(t), \tilde{\rho}_L(0)\}\rangle$. Therefore, the standard bozonization tools can be readily applied. Within the bosonization framework, the effect of the electron-electron interaction in the central part of the wire is to induce the scattering of density fluctuations at the boundaries of the interaction region. We will assume in the following that the characteristic bosonic momenta (set in our problem by the the voltage U in the distribution function $n_R(\epsilon)$) are small compared to the inverse length of the transition region between the interacting and non-interacting parts of the wire. The bosonic reflection at the wire boundary is then characterized[21] by the (momentum independent) amplitude

$$b = (1 - K)/(1 + K) \tag{4}$$

and the density $\tilde{\rho}_L$ can be expressed in terms of those at the sources SR and SL via

$$\tilde{\rho}_L(t) = (1 - b^2) \sum_{n=0}^{\infty} b^{2n} \rho_L(t - (2n+1)\tau_l)$$

$$+ b\rho_R(t) - (1 - b^2) \sum_{n=0}^{\infty} b^{2n+1} \rho_R(t - (2n+2)\tau_l). \tag{5}$$

Here $\tau_l = Kl/u$ is the time needed to a density perturbation to cross the LL wire.

Evaluating now the correlation function of $\tilde{\rho}_L$ with the account of the fermionic distribution functions in the leads SR and SL, one finds

$$S_{D2}^{(SL)}(\omega) = \frac{(1 - b^2)^2 S_0(\omega)}{|1 - b^2 e^{2i\omega\tau_l}|^2} + \frac{2b^2 [1 - \cos(2\omega\tau_l)] S_R(\omega)}{|1 - b^2 e^{2i\omega\tau_l}|^2}, \tag{6}$$

where $S_R(\omega)$ is the noise of non-interacting electrons with the double-step distribution (1),

$$S_R(\omega) = \frac{e^2}{2\pi} [|\omega| + 2h(1 - h)(U - |\omega|)\Theta(U - |\omega|)]. \tag{}$$

Due to charge conservation and the absence of fermionic backscattering in the wire the zero frequency component of $\tilde{\rho}_L$ in Eq. (5) coincides with that of the incoming density ρ_L. As a consequence, $S_{D2}^{(SL)}(\omega)$ is insensitive to the interaction strength for low frequencies $\omega \ll 1/\tau_l$ and vanishes at $\omega = 0$.

Both contributions $S_{D2}^{(S2)}(\omega)$ and $S_{D2}^{(SL)}(\omega)$ that we have evaluated up to now are thus interaction-independent in the low-frequency regime and vanish at zero frequency. We turn now to the analysis of the remaining cross-correlation term, $S_{D2}^{(\text{x})}(\omega)$. We will show that it is non-trivial and interaction-sensitive in the limit of $\omega = 0$. In time domain, this contribution can be presented in the form

$$S_{D2}^{(\text{x})}(\tau, V) = 2e^2u^2 \operatorname{Re}\left[G_{S2}^<(-\tau)G_{\tilde{L}}^>(\tau) + G_{S2}^>(-\tau)G_{\tilde{L}}^<(\tau) \right]. \tag{7}$$

Here $G_{S2}^{\gtrless}(t) = -e^{-iVt}/2\pi u(t \mp i0)$ stand for the time-domain Keldysh Green functions of Ψ_{S2}, while $G_{\tilde{L}}^{\gtrless}$ are Green functions of left-moving electrons leaving the interacting wire. It is not difficult to check that the zero-frequency noise is directly related to the distribution function of $\Psi_{\tilde{L}}$ electrons, $n_{\tilde{L}}(\epsilon)$, via

$$\partial_V S_{D2}^{(\text{x})}(\omega = 0, V) = \frac{e^2}{\pi}[1 - 2n_{\tilde{L}}(V)]. \tag{8}$$

The non-equilibrium bosonization technique for evaluation of correlation functions in a far-from-equilibrium LL in the framework of Keldysh formalism was developed in Refs. 25,31,32. It was shown, that arbitrary (including many-particle) electronic correlation function can be expressed as a product of Fredholm determinants, $G \sim \Delta_R[\delta_R(t)]\Delta_L[\delta_L(t)]$, having the form

$$\Delta_\eta[\delta_\eta(t)] = \det\left[1 + \left(e^{i\delta_\eta(t)} - 1 \right) n_\eta(\epsilon) \right]. \tag{9}$$

Here, $n_{R(L)}(\epsilon)$ denotes the distribution function of electrons injected into the LL from the right (left) lead, while the phases $\delta_\eta(t)$ encode the information on the correlation function of interest and on the interaction strength in the wire. The time t and energy ϵ in Eq.(9) are understood as canonically conjugate variables, $\epsilon = i\hbar(\partial/\partial t)$. Fredholm determinants of a type similar to Eq. (9) arise also in the theory of full counting statistics[33], non-equilibrium Fermi-edge singularity[34], chiral 1D systems (including quantum Hall
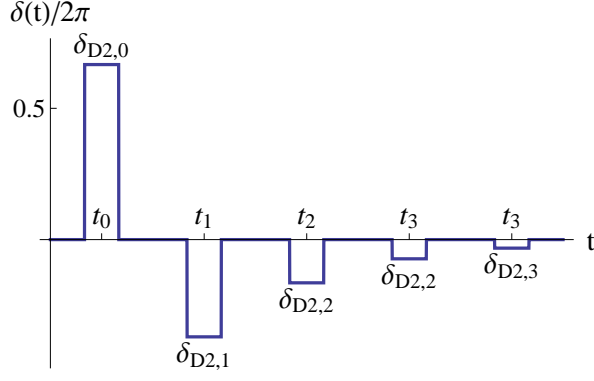
FIG. 2: Phase $\delta(t)$ determining the charge noise at the drain D2, see Eqs. (11), (15). The phases $\delta_{D2,n}$ are given by Eq. (13). We have assumed $K = 0.2$ to generate the plot.

Mach-Zehnder interferometry)[35], and spectral functions of nonlinear Luttinger liquids[36].

Evaluating the single-particle correlation functions of the electrons leaving the LL [which are involved in Eq.(7)] by means of this technique and taking into account the zero-temperature electronic distribution in the source $S2$ (see Fig. 1), we get

$$G_{\bar{L}}^{\gtrless}(\tau) = -\frac{1}{2\pi u(\tau \mp i0)}\bar{\Delta}(\tau) \qquad (10)$$

Here $\bar{\Delta}(\tau)$ stands for the Fredholm determinant (9) normalized to its zero-temperature value

$$\bar{\Delta}(\tau) = \frac{\det\left[1 + \left(e^{i\delta(t)} - 1\right)n_R(\epsilon)\right]}{\det\left[1 + \left(e^{i\delta(t)} - 1\right)n_0(\epsilon)\right]}, \qquad (11)$$

with the phase $\delta(t)$ being a sequence of rectangular-shaped pulses of duration τ centered at $t_n = 2n\tau_l$, $n = 0, 1, \ldots$ (see Fig. 2):

$$\delta(t) = \sum_{n=0}^{\infty} \delta_{D2,n}\Theta(t - t_n - \tau/2)\Theta(-t + t_n + \tau/2), \quad (12)$$

$$\delta_{D2,n} = 2\pi \begin{cases} b & \text{for } n = 0 \\ -(1 - b^2)b^{2n-1} & \text{for } n > 0 \end{cases}. \qquad (13)$$

Substitution of Eq. (10) into Eq. (7) yields

$$S_{D2}^{(x)}(\omega, V) = 2S_0(\omega)$$
$$- \frac{e^2}{\pi^2}\int d\tau \, \frac{\text{Re}\left(\bar{\Delta}(\tau)e^{iV\tau} - 1\right)}{\tau^2}\cos(\omega\tau). \quad (14)$$

In the low-frequency limit, we thus obtain for the noise of electrons at the drain D2:

$$S_{D2}(\omega = 0, V) = -\frac{e^2|t|^2|r|^2}{\pi^2}\int d\tau \, \frac{\text{Re}\left(\bar{\Delta}(\tau)e^{ieV\tau} - 1\right)}{\tau^2}. \qquad (15)$$

We proceed now to the analysis of the result (15). The τ-dependence of the normalized determinant $\bar{\Delta}(\tau)$

is characterized by two distinct time scales. First, there is the scale $\tau_l = Kl/u$ set by the length of the interacting wire which determines the time interval between successive pulses in Eq. (12). Under the assumption $\tau \ll \tau_l$ we can approximate the determinant $\bar{\Delta}(\tau)$ by a product of Toeplitz determinants corresponding to the individual rectangular-shaped pulses in Eq. (12),

$$\bar{\Delta}(\tau) = \prod_{n=0}^{\infty} \bar{\Delta}(\tau, \delta_n). \qquad (16)$$

The second scale controls the long-time behavior of Eq. (16). According to the asymptotic theory of Toeplitz determinants reviewed in Refs. 32,37 the long-time asymptotics of $\bar{\Delta}(\tau)$ is controlled by the exponential decay, $\bar{\Delta}(\tau) \propto e^{-\tau/2\tau_\phi}$, with a non-equilibrium dephasing rate given by

$$\frac{1}{\tau_\phi} = -\frac{U}{\pi}\sum_{n=0}^{\infty} \text{Re}\ln\left[1 + (e^{-i\delta_n} - 1)h\right]. \qquad (17)$$

For the generic parameters, the dephasing rate (17) is of the order of U, thus setting a characteristic time scale $1/U$. Therefore, the integral (15) converges at times τ set[38] by $1/\max\{U, V\}$. We thus conclude that the approximation Eq. (16) for the determinant entering Eq. (15) can be safely applied provided that at least one of voltages U and V is large compared to the inverse flight time $1/\tau_l$.

B. Asymptotic behavior

Equations (15) and (16) render the current noise $S_{2D}(\omega = 0, V)$ amiable to straightforward numerical evaluation (see, e.g., Ref. 39 for a detailed account of numerical procedure). Two limiting cases can also be studied analytically. First, in the weak-interaction regime, $|K - 1| \ll 1$, all the phases $\delta_{D2,n}$ are small, enabling perturbative evaluation of the determinant $\bar{\Delta}(\tau)$. The result reads

$$\ln\bar{\Delta}(\tau) = -\alpha_1\int_0^U d\omega \frac{\sin^2\omega\tau/2}{\omega^2}(U - \omega), \quad (18)$$

$$\alpha_1 = 2(K - 1)^2 h(1 - h). \qquad (19)$$

with the asymptotic behavior

$$\ln\bar{\Delta}(\tau) = -\frac{\pi\alpha_1 U\tau}{4} + \frac{\alpha_1}{2}\ln U\tau, \qquad U\tau \gg 1. \quad (20)$$

Examining the integral (15), we see that at $V \gg \alpha_1 U$ we can further expand $\bar{\Delta}(\tau) = \exp\left(\ln\bar{\Delta}(\tau)\right)$ in powers of α_1. On the other hand, for $V \ll \alpha_1 U$ the integral (15) is dominated by long times where the asymptotics (20) applies. We thus obtain the following result for the noise in the case of a weak interaction:

$$S_{D2}(\omega = 0, V) \underset{|K-1|\ll 1}{=} \frac{e^2|t|^2|r|^2 U}{\pi}f(\alpha_1, V/U), \quad (21)$$

with the small parameter α_1 given by Eq. (19) and with a dimensionless function f given by

$$f(\alpha, x) =$$
$$= \begin{cases} |x|, & x > 1 \\ |x| + \alpha \left[|x| - 1 - \frac{1+|x|}{2} \ln |x| \right], & \alpha \ll |x| < 1 \\ -\frac{\alpha}{2} + \frac{2}{\pi} x \arctan \frac{4x}{\pi\alpha} - \frac{\alpha}{4} \ln \left(x^2 + \frac{\pi^2 \alpha^2}{16} \right), & |x| \lesssim \alpha. \end{cases}$$
(22)

The second limit that can be treated fully analytically is that of a strong repulsive interaction, $K \ll 1$. In this situation, the bosonic reflection coefficient is close to 1. As a consequence, all but the first phase differences $\delta_{D2,n>0}$ are small, while $\delta_{D2,0}$ is close to 2π. The determinant $\bar{\Delta}(\tau, \delta)$ at the phase $\delta = 2\pi$ corresponds to free fermions and can be computed exactly via the refermionization procedure,

$$\bar{\Delta}(\tau, 2\pi) = (1-h)e^{-i\epsilon_0 \tau} + h e^{-i\epsilon_1 \tau}. \quad (23)$$

Treating all but the first factors in Eq. (16) perturbatively [cf. Eqs. (18) and (19)] and also taking into account the perturbative correction to the zeroth-order approximation (23) for the determinant $\bar{\Delta}(\tau, \delta_{D2,0})$, we arrive at

$$\bar{\Delta}(\tau) = \left[\bar{\Delta}(\tau, 2\pi) - \frac{\alpha_0}{2} \left(e^{-i\epsilon_0 \tau} g(U\tau) + e^{-i\epsilon_1 \tau} g(-U\tau) \right) \right]$$
$$\times \exp \left[-\alpha_0 \int_0^U d\omega \frac{\sin^2 \omega\tau/2}{\omega^2} (U - \omega) \right] \quad (24)$$

with

$$\alpha_0 = 8Kh(1-h), \quad (25)$$
$$g(x) = \gamma - \mathrm{Ci}(x) + \ln x + i\,\mathrm{Si}(x). \quad (26)$$

Here γ is the Euler constant, while $\mathrm{Ci}(x)$ and $\mathrm{Si}(x)$ stand for the integral cosine and sine functions.

To zeroth order in the small parameter α_0, Eq. (25), we get

$$S_{D2}(\omega = 0, V) \underset{K=0}{=} \frac{e^2 |t|^2 |r|^2}{\pi} \left[(1-h)|V - \epsilon_0| + h|V - \epsilon_1| \right]. \quad (27)$$

An analysis analogous to the one that leaded to Eqs. (21) and (22) allows us to establish also the first correction to Eq. (27), yielding

$$S_{D2}(\omega = 0, V) \underset{K \to 0}{=} \frac{e^2 U |t|^2 |r|^2}{\pi}$$
$$\times \left\{ (1-h) f \left(\alpha_0, \frac{V - \epsilon_0}{U} \right) + h f \left(\alpha_0, \frac{V - \epsilon_1}{U} \right) \right.$$
$$\left. + \alpha_0 \left[p \left(\frac{V - \epsilon_0}{U} \right) + p \left(\frac{\epsilon_1 - V}{U} \right) \right] \Theta(V > \epsilon_0) \Theta(V < \epsilon_1) \right\}. \quad (28)$$

with $p(x) \equiv x \ln x$ and α_0 given by Eq. (25).
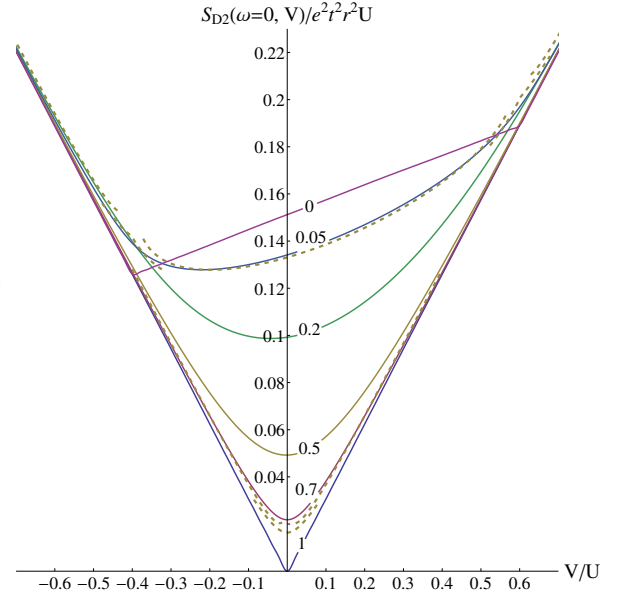

FIG. 3: Zero-frequency noise at the drain D2, $S_{D2}(\omega = 0, V)$, given by Eq. (15), as a function of the voltage ratio V/U for different values of the interaction strength. The parameter h is chosen to be 0.4. The curves are labeled according to the LL parameter K characterizing the interaction strength. Dashed curves represent appropriate asymptotic expressions [Eqs. (21), (22) for a relatively weak interaction, $K = 0.7$, and Eqs. (28), (22) for a strong interaction, $K = 0.05$].
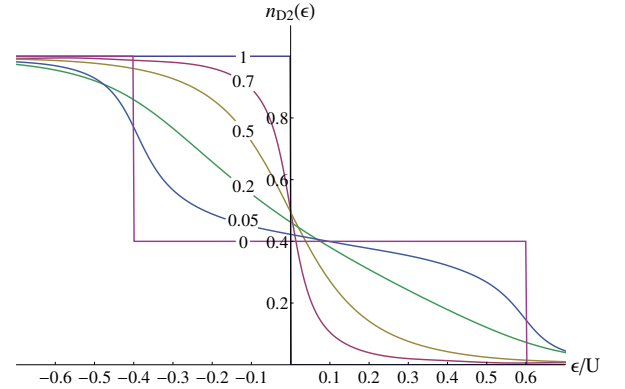

FIG. 4: Distribution of electrons over energies at the drain D2, $n_{D2}(\epsilon)$, for different values of the interaction strength. The parameter h is chosen to be 0.4. The curves are labeled according to the LL parameter K characterizing the interaction strength.

C. Numerical evaluation

We turn now to the case of a generic interaction strength, when the determinant in Eqs. (15) and (16) is computed numerically. Figures 3 and 4 present the resulting evolution of the noise $S_{D2}(\omega = 0, V)$ [as given by Eq. (15)] and of the distribution function $n_{D2}(\epsilon)$ upon variation of the LL parameter K characterizing the interaction strength. We have set $h = 0.4$ to generate
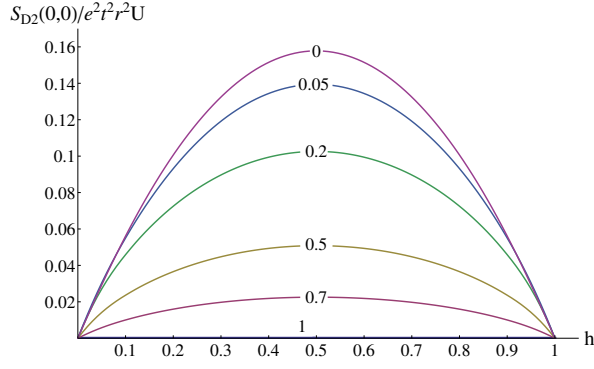
FIG. 5: Noise at the drain D2 at zero frequency and zero voltage V, $S_{D2}(\omega = 0, V = 0)$, as a function of the step height h for different values of the interaction strength. The curves are labeled according to the LL parameter K characterizing the interaction strength.
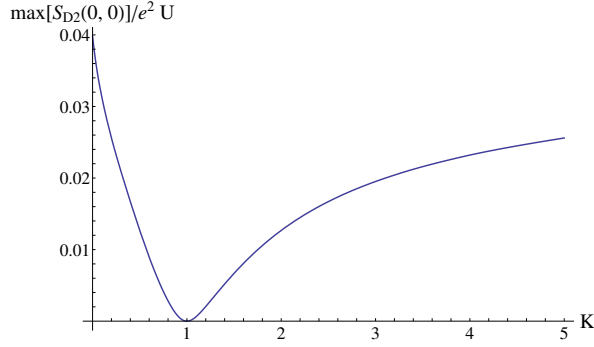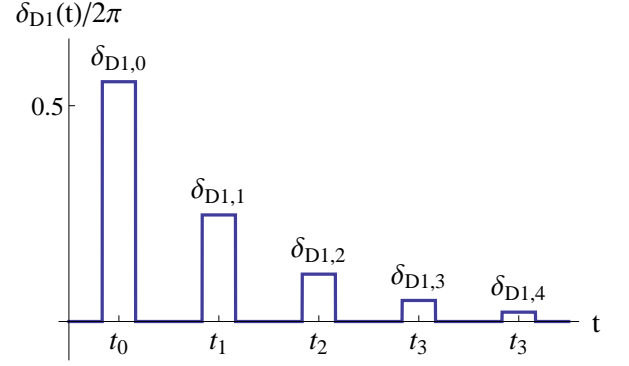

FIG. 7: Phase $\delta(t)$ determining the charge noise at the drain D1, see Eqs. (11), (30). The phases $\delta_{D1,n}$ are given by Eq. (32). We have assumed $K = 0.2$ to generate the plot.
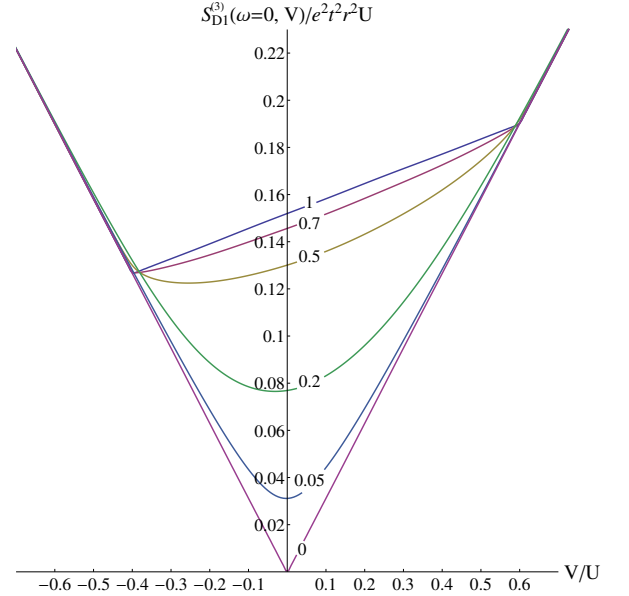

FIG. 6: The maximal noise in drain D2 at zero frequency and zero voltage V, $\max_{h,r^2,t^2} S_{D2}(\omega = 0, V = 0)$, as a function of the Luttinger liquid parameter K. The noise attains maximum when the initial distribution is particle-hole symmetric ($h = 0.5$) and the QPC mixing the electrons from the LL wire with those from the source S2 has reflection probability $r^2 = t^2 = 0.5$, see Sec. III C. The maximal noise equals $1/8\pi$ at $K = 0$ and approaches the same value asymptotically as $K \to \infty$ (see Sec. V for the discussion of the case of attractive interaction $K > 1$).

FIG. 8: Zero-frequency noise at the drain D1, $S_{D1}^{(x)}(\omega = 0, V)$, as a function of the voltage V for different values of the interaction strength. The parameter h is chosen to be 0.4. The curves are labeled according to the LL parameter K characterizing the interaction strength.

the plots. The dashed curves provide the comparison to the appropriate asymptotic expressions [(21) for $K = 0.7$ and (28) for $K = 0.05$]. Upon increase of the interaction strength, the distribution function of outgoing electrons n_{D2} evolves from the zero-temperature distribution of the incoming left-moving electrons towards the double-step distribution of the electrons at source SR. It is interesting to note that the characteristic width of the distribution function is non-monotonous as a function of the interaction strength, which is related to the non-monotonous dependence of the dephasing rate (17) on the Luttinger parameter K.

Figure 5 demonstrates the dependence of the noise at zero voltage V on the parameter h of the double-step distribution (1) of incoming right-moving electrons. The step height h can be experimentally varied by changing

the transmission of the QPC0 creating the initial distribution. The noise attains the maximal value when the initial distribution is particle-hole symmetric ($h = 0.5$) and the QPC mixing the electrons from the LL wire with those from the source S2 has reflection probability $r^2 = t^2 = 0.5$. The ration of this maximal noise to the voltage U is a universal function of the LL parameter. Figures 3, 5 and 6 show that the current noise $S_{D2}(\omega = 0)$ provides a direct access to the value of the LL parameter K.
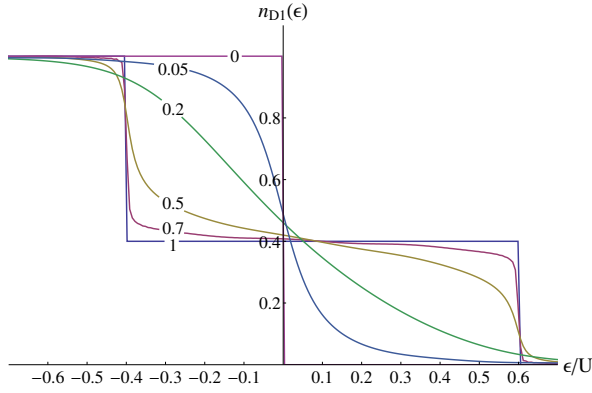
FIG. 9: Distribution of the electrons over energies at the drain D1, $n_{D1}(\epsilon)$, for different values of the interaction strength. The parameter h is chosen to be 0.4. The curves are labeled according to the LL parameter K characterizing the interaction strength.

IV. NOISE AT DRAIN D1

Let us now consider briefly the noise at the drain D1. In complete analogy to S_{D2}, it can be written as

$$S_{D1}(\omega, V) = |r|^4 S_{D1}^{(S1)} + |t|^4 S_{D1}^{(SR)} + |r|^2 |t|^2 S_{D1}^{(x)}. \quad (29)$$

In contrast to the case of the noise at D2 the term $S_{D1}^{(SR)}(\omega)$ does not vanish at zero frequency, $S_{D1}^{(SR)}(\omega = 0) = e^2 h(1-h)U/\pi$. However, in full similarity with the D2 noise, only the last contribution in Eq. (29) is sensitive to the voltage V at the QPC. Concentrating on the V-dependence of the noise, we thus observe that Eq. (15) applies,

$$S_{D1}^{(x)}(\omega = 0, V) = -\frac{e^2}{\pi^2} \int d\tau \, \frac{\text{Re}\left(\bar{\Delta}(\tau)e^{ieV\tau} - 1\right)}{\tau^2}, \quad (30)$$

where the phase $\delta(t)$ is now given by (see Fig. 7)

$$\delta(t) = \sum_{n=0}^{\infty} \delta_{D1,n}\Theta(t - t_n - \tau/2)\Theta(-t + t_n + \tau/2) \quad (31)$$

$$\delta_{D1,n} = 2\pi(1-b^2)b^{2n}. \quad (32)$$

Figures 8 and 9 show results of a numerical evaluation of the noise $S_{D1}^{(x)}$ and the distribution of the electrons at drain D1. The perturbative analysis of the D1 noise in the weak- and strong-interaction limits can be obtained by a straightforward generalization of the corresponding results for the noise D2 presented above. We reiterate that although the D1 noise contains an additional contribution $S_{D1}^{(SR)}$, this contribution is independent on the voltage V. Therefore, the V dependence of the D1 noise can also be used (along with the D2 noise) for extracting the LL interaction parameter K.

V. ATTRACTIVE INTERACTION

So far [and in particular in the discussion of the strong interaction limit of the model, Eq. (28)] we were mostly concentrating on the situation of repulsive interaction in the system, $K < 1$. The extension of our results to the case of attractive interaction, $K > 1$ (particularly relevant in the context of superconducting wires[3], cold atoms[13] and fractional quantum Hall systems[17]), is straightforward due to the symmetry of the bosonic reflection amplitude b, Eq. (4),

$$b(1/K) = -b(K). \quad (33)$$

Equation (33) implies that the charge noise $S_{D1}(\omega, V)$ at drain D1 is invariant with respect to the transformation $K \to 1/K$, while the noise and the distribution function at D2 obey

$$S_{D2}(\omega, V, 1/K) = S_{D2}(\omega, -V, K), \quad (34)$$

$$\tilde{n}_L(\epsilon, 1/K) = 1 - \tilde{n}_L(-\epsilon, K). \quad (35)$$

In particular, in the limit of infinitely strong attractive interaction, $K = \infty$,

$$\tilde{n}_L(\epsilon) = 1 - n_R(-\epsilon) \quad (36)$$

due to Andreev reflection of electrons on the boundary of interaction region.

VI. SUMMARY

To summarize, we have studied the low-frequency noise of interacting electrons in a 1D structure with counterpropagating modes (quantum wire), assuming a single channel in each direction. Experimental realizations of such structures include also artificial quantum wires formed by counter propagating quantum Hall channels coupled by the interaction. The system is driven out of equilibrium by a QPC0 with an applied voltage, which induces a double-step energy distribution of incoming electrons on one side of the device. A second QPC serves to explore the statistics of outgoing electrons. We evaluate the dependence of the zero-frequency noise on K and on parameters of both QPCs (transparencies and voltages).

Our general result, Eq. (15), expresses the noise in the drain D2 in terms of a Fredholm determinant $\bar{\Delta}(\tau)$. In the limits of weak and strong interaction analytical asymptotic (21) and (28) have been obtained. For a generic interaction strength, the noise can be readily evaluated numerically, as shown in Figs. 3, 5, 6. Similar results hold for the noise in the drain D1. Our findings demonstrate that measurement of a low-frequency noise in such a setup allows one to extract the information about the Luttinger liquid constant K which is the key parameter characterizing an interacting 1D system.

Upon completion of this work we have learned about a related activity on the noise in systems of co-propagating channels[40].

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